The Granular Leidenfrost Effect

Density inversion in vibrofluidized granular matter

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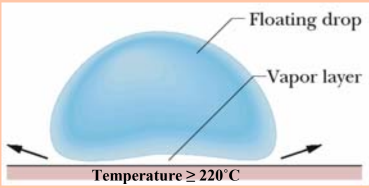
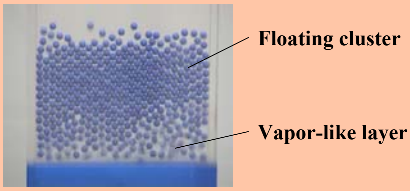
Johann Gottlob Leidenfrost (1756)



Leidenfrost first discovered that a drop of water on a red-hot spoon floats on its own vapor layer. The effect occurs above a critical temperature of 220°C. The drop survives for an astonishingly long time (easily exceeding a minute) because the vapor-cushion prevents direct heat transfer from the hot surface to the droplet.

Granular Leidenfrost effect



The *granular* Leidenfrost effect occurs when glass beads are vertically vibrated above a critical shaking strength. A liquid-like cluster is elevated and supported by just a few fast particles underneath.

Just as in the original Leidenfrost effect, the density is inverted.

Inversion height

The density ρ is experimentally determined from high-speed recordings (@1000fps) by counting the number of black pixels in each row. The density profile shown here is obtained by averaging over 300 frames. The density inversion is clearly visible. At the *inversion height* h_{inv} the derivative ρ becomes zero.

The *four* natural dimensionless numbers for the granular Leidenfrost effect are:

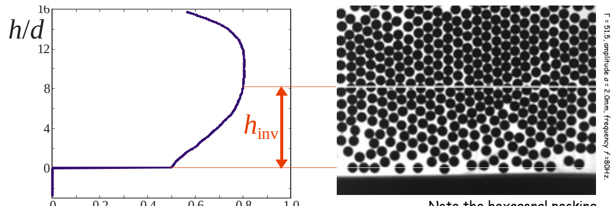
[1] Dimensionless shaking strength $\Gamma = \dfrac{a(2\pi f)^2}{g}$

[2] Amplitude of the vibration $\dfrac{a}{d}$

[3] Inelasticity of the collisions $(1-e^2)$

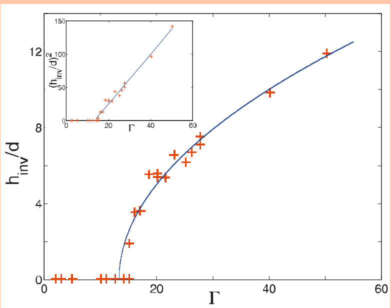
[4] Number of layers F

Density vs. height (in bead diameters d) for F = 16 layers of beads.



Note the hexagonal packing in the floating cluster

Second order phase transition

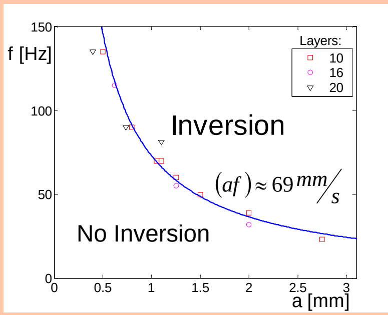


The inversion height h_{inv} shows a continuous, second order phase transition as function of the shaking strength Γ.

← For F = 10 layers of beads the transition takes place at a critical shaking strength Γ_c = 15. The inset shows that $h_{inv} \propto (\Gamma - \Gamma_c)^{1/2}$, the standard mean-field power law behavior.

Inversion is found when (af) exceeds a critical value of 69mm/s. →

We find that a minimum number of F = 10 layers is needed for density inversion. For less beads no floating cluster forms and the density is a decreasing function of height.



Dimensionless control parameters

We describe the vibrated grains by three hydrodynamic equations:

Equation of state $$p = nT\frac{n_c + n}{n_c - n}$$

Here p is the pressure, n the number density (with $n_c = 2/d^2\sqrt{3}$ the maximal value corresponding to hexagonal close packing) and $T \propto m\langle v^2\rangle$ the granular temperature. In the dilute limit ($n/n_c \ll 1$) the equation reduces to the ideal gas law, but for larger densities excluded volume effects become dominant.

Force balance $$\frac{dp}{dz} = -mgn$$

The pressure difference over a slice of height dz balances the weight of the particles in that slice.

Energy balance $$\frac{d}{dz}\left(AnlT^{1/2}\frac{dT}{dz} + (1-e^2)lT^{3/2}\frac{dn}{dz}\right) = B(1-e^2)\frac{nT^{3/2}}{l}$$

The left term is the energy flux, inserted into the system by the vibrating bottom, which in the steady state is balanced by the energy dissipation via inelastic collisions (right term). A and B follow from kinetic theory: $A \approx 1.94 + 0.51(1-e^2)$, $B \approx 0.86$ (with e the coefficient of normal restitution, 0.95 for our glass beads) and l is the mean free path of the particles:

$$l = \frac{1}{\sqrt{8}nd}\,\frac{n_c - n}{n_c - 0.39n}$$

Non-dimensionalizing these equations we find that the problem is governed by only *three* dimensionless combinations:

[1] The energy input $S = \dfrac{T_0}{mgd}$ = the kinetic energy of the particles at the bottom (T_0), divided by the potential energy needed for a particle to overcome 1 layer.

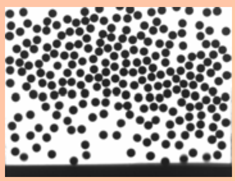
Since $T_0 \propto (af)^2$ [the squared bottom velocity], S is the *product* of shaking strength $\Gamma \propto af^2$ and vibration amplitude a/d combined into one dimensionless parameter. This is in agreement with the experimental fact that inversion occurs for $(af) > 69$ mm/s.

[2] The inelasticity $(1-e^2)$ = 0.098 for our glass beads

Finally, from the conservation of particles:

[3] Number of layers F

A minimum number of layers is required to get density inversion, both in theory and experiment. The snapshot shows the situation for the threshold value of F = 10 layers.



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