Granular Leidenfrost Effect: Experiment and Theory of Floating Particle Clusters

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Granular material is vertically vibrated in a 2D container: above a critical shaking strength, and for a sufficient number of beads, a crystalline cluster is elevated and supported by a dilute gaseous layer of fast beads underneath. We call this phenomenon the granular Leidenfrost effect. The experimental observations are explained by a hydrodynamic model featuring three dimensionless control parameters: the energy input S, the number of particle layers F, and the inelasticity of the particle collisions ε. The (S, F) phase diagram, in which the Leidenfrost state lies between the purely solid and gas phases, shows accurate agreement between experiment and theory.

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Vertically shaken granular matter typically exhibits a region of reduced density just above the vibrating bottom [1–5]. An exceptionally strong form of this so-called density inversion was recently encountered in a theoretical study by Meerson *et al.* [6]: for sufficiently strong shaking a dense cluster of particles, showing a hexagonal packing, was observed to be elevated and supported by a dilute layer of fast particles underneath.

Here we present the first *experimental* observation of this phenomenon, which we will call the granular Leidenfrost effect. It is analogous to the original Leidenfrost effect of a water droplet hovering over a hot plate [7,8]: when the temperature of the plate exceeds the Leidenfrost temperature $T_L \approx 220\,^{\circ}\mathrm{C}$ (equivalent to the critical shaking strength in the granular system), the bottom layer of the drop vaporizes instantly and prevents direct heat transfer from the plate to the drop, causing the droplet to hover and survive for a long time.

We also give a *theoretical* explanation in the spirit of Meerson *et al.* [6,9,10]. These authors focused on the point where the density at the bottom first becomes inverted, which is a precursor to the granular Leidenfrost effect (not yet the actual phase separation). We study the subsequent transition from this density-inverted state to the Leidenfrost state in which the solid and gas phases coexist. A major challenge in granular research today is to achieve a hydrodynamiclike continuum description [11–16], which, however, in many cases breaks down due to the tendency of the particles to cluster together [17,18]. We show that the Leidenfrost effect (despite the clustered phase) is well described by a hydrodynamic model.

Our experimental setup (Fig. 1) consists of a quasi-2D container ($10 \times 0.45 \times 14$ cm) [19] filled with glass beads of diameter $d = 4.0$ mm, density $\rho = 2.5\ g/\mathrm{cm}^3$, and coefficient of normal restitution $e \approx 0.95$. The setup is mounted on a shaker with tunable frequency f and amplitude a. The Leidenfrost effect, see Fig. 1, is stably reproduced for given, sufficiently large values of the shaking strength and the number of particle layers.

The four natural dimensionless control parameters to analyze the experiment are (i) the shaking acceleration (with g the gravitational acceleration):

$$\Gamma = \frac{a(2\pi f)^2}{g}, \tag{1}$$

(ii) the number of bead layers F, (iii) the dimensionless shaking amplitude $A = a/d$, and (iv) the inelasticity parameter $\varepsilon = (1 - e^2)$.

First the dependence on Γ is investigated for a fixed number of layers $F = 16$. Figures 2(a) and 2(b) show an experimental snapshot and the corresponding density profile $n(y)$ (determined by counting the number of black pixels in each horizontal row) at moderate shaking, $\Gamma = 7.7$. The snapshot shows a hexagonal packing and this is reflected in the periodic structure of $n(y)$; i.e., the particles behave like a solid crystal. The theoretical profile in Fig. 2(c) does not show this periodicity, reflecting the continuum (nonparticulate) character of the model.

At vigorous shaking [Figs. 2(d) and 2(e)] the Leidenfrost state is observed: a crystalline cluster floats on top of a

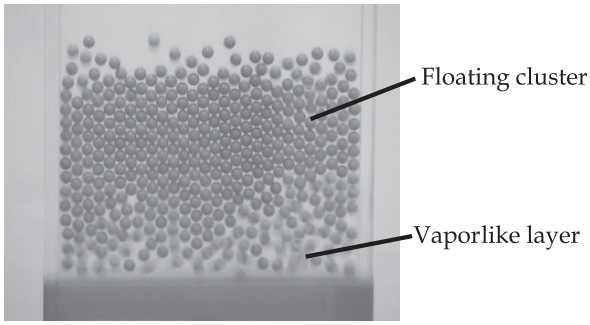

FIG. 1. Granular Leidenfrost effect: glass beads, vertically vibrated above a critical shaking strength, form a crystalline cluster that is elevated and supported by a vaporlike layer of fast particles underneath. The thickness of the dilute layer oscillates in time (never vanishing) due to the motion of the bottom, while the cluster floats steadily at the same position.

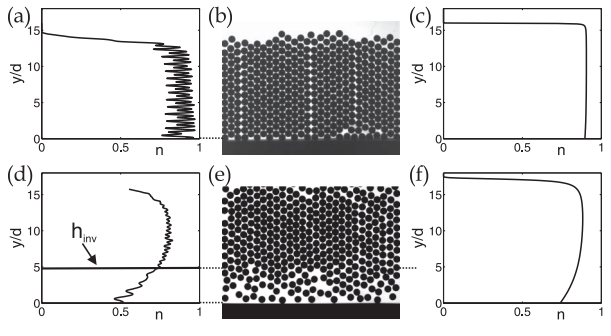

FIG. 2. (a) Density profile, averaged over 300 consecutive snap shots, as a function of height for $F = 16$ layers at shaking strength $\Gamma = 7.7$ ($a = 0.3$ mm, $f = 80$ Hz). The origin $y/d = 0$ is set at the maximal positive displacement of the vibrating bottom. (b) A typical snapshot of this experiment recorded by a high-speed camera (1000 fps) and (c) the theoretical profile from the model in Eqs. (4)–(6) with $S = 4.0$. (d) Density profile for $F = 16$ layers at $\Gamma = 51.5$ ($a = 2.0$ mm, $f = 80$ Hz), showing the Leidenfrost state. The inversion height h_{inv} marks the border between the gaseous and the solid phase; it is determined via the method illustrated in Fig. 3. (e) The corresponding experimental snapshot and (f) the theoretical profile for $S = 80$.

dilute gaseous layer. The particular experiment of Fig. 2(e) was performed at $\Gamma = 51.5$, well above the critical shaking strength ($\Gamma_c \approx 25$ for $F = 16$ layers) at which the Leidenfrost effect sets in. Increasing the shaking strength even more causes the crystalline layer to become thinner and more dilute, until (at some very high value of Γ, beyond the capacity of our shaker) it will disappear altogether and the whole system becomes gaseous.

Regarding the dependence on the second control parameter (F), a Leidenfrost state only occurs for $F \gtrsim 8$ particle layers. For smaller F one witnesses a direct transition from the solid phase to a pure granular gas.

Now, what is a suitable order parameter to distinguish between the gaseous and the crystalline phase in the Leidenfrost state? To answer this, we turn to the pair correlations of the particles in a horizontal strip $(y, y + dy)$ (where dy is chosen to be equal to a particle diameter):

$$g_y(x) = \frac{1}{N} \sum_{i,j \ in \ (y,y+dy)} \sum_{i \neq j} \delta(x - (x_i - x_j)), \qquad (2)$$

with N the number of particles in the strip, and $x_{i,j}$ their horizontal positions. Figure 3(b) gives $g_y(x)$ for a typical gaslike strip near the bottom, where the particle positions show hardly any correlation. On the other hand, the strip inside the floating cluster of Fig. 3(c) shows a strong periodic, crystalline correlation. This clear distinction between periodic and nonperiodic behavior is exploited in the order parameter O, which we define as the modulus of the integrated difference between $g_y(x)$ and its running mean: O is the shaded area in Figs. 3(b) and 3(c). Figure 3(d) shows O as a function of height y, exhibiting a clear transition from the gaslike to the crystalline phase. By making a fit through $O(y)$ in which we smoothen out the

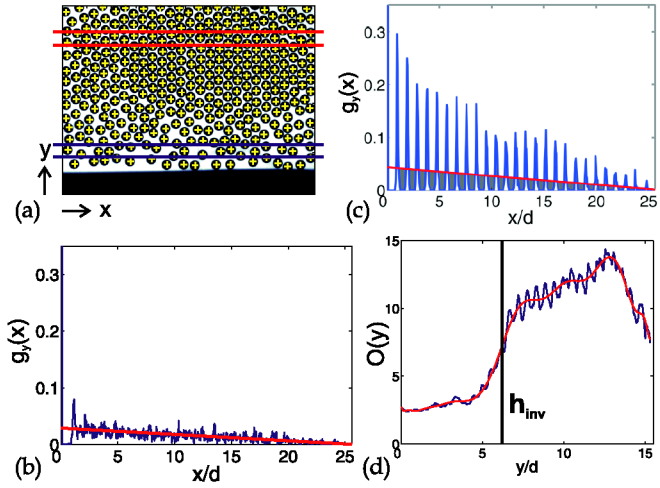

FIG. 3 (color online). (a) Experimental Leidenfrost state for $\Gamma = 64.4$ ($a = 2.5$ mm, $f = 80$ Hz), with $F = 16$ layers: two horizontal strips have been selected, one in the gaseous layer and one in the crystalline region. (b) The correlation $g_y(x)$ between the particle centers (+) in the gaseous strip, determined via Eq. (2). (c) The same in the crystalline strip, showing a clear periodicity. The shaded area is the order parameter O. (d) $O(y)$ (in arbitrary area units), determined for a number of strips at different heights. The smoothed fit through $O(y)$ is used to determine the inversion height h_{inv}, marking the transition from gas to crystal.

oscillations associated with the crystalline order, the inversion height h_{inv} can be determined as the point where the slope of the fit is maximal [see Fig. 3(d)].

For all experimental runs, we determined from the associated order parameter plots [as in Fig. 3(d)] the critical shaking amplitude a and frequency f at the transition to the Leidenfrost state, i.e., when the inversion height becomes nonzero for the first time. The result is plotted in Fig. 4. Along the curve in the (a, f) plane that marks the transition towards the Leidenfrost state [Fig. 4(a)], the product af is constant, or equivalently:

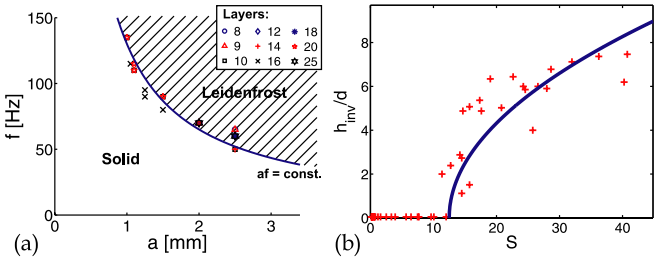

FIG. 4 (color online). (a) The critical values of the shaking amplitude a and frequency f at which the transition from the solid to the Leidenfrost state occurs, for a number of experiments with $F = 8, \ldots, 25$ layers. The product af is constant along the transition curve, or equivalently, $S \equiv \Gamma A \propto (af)^2$ is constant. (b) The inversion height h_{inv} vs the shaking parameter S for $F = 16$ layers, indicating that the transition is a continuous, second-order phase transition. The fit through the experimental data is of the form $h_{\mathrm{inv}}/d \propto (S - S_{\mathrm{onset}})^{1/2}$.

$$S \equiv \Gamma A = \frac{(a 2\pi f)^2}{g d} \tag{3}$$

is constant at the transition. So S (and not Γ) is the fundamental shaking parameter in the experiments. We will see later that this is not only true in our experiments, but also in the theoretical model.

Therefore, in Fig. 5 we present the location of the Leidenfrost regime, marked by crosses (×), in the (S, F) plane and not the (Γ, F) plane. The transition from the solid phase (●) to the Leidenfrost regime occurs along a nearly horizontal line, i.e., constant S, in agreement with the result from Fig. 4. Increasing the shaking strength further, the Leidenfrost state disappears again and the system now behaves as a pure gas (○). In experiment, this last transition can only be observed around the critical number of $F \approx 8$ layers; the transition line rises so steeply that for more layers our shaker is not strong enough to vaporize the Leidenfrost state.

To explain the experimental observations theoretically, we use a continuum description of the granular material. It is to be regarded as a minimal model (disregarding the effect of the sidewalls, which makes it essentially one dimensional, and sidestepping the particle nature of the system), not intended to capture all the details of the experimental system, but to explain the granular Leidenfrost effect. The model is based on three hydrodynamic equations that have been derived within the context of the kinetic theory of granular gases [6,12,13,15].

The first one is the standard force balance:

$$\frac{dp}{dy} = -mgn, \tag{4}$$

with $p(y)$ the pressure, m the mass of a single particle, and $n(y)$ the number density.

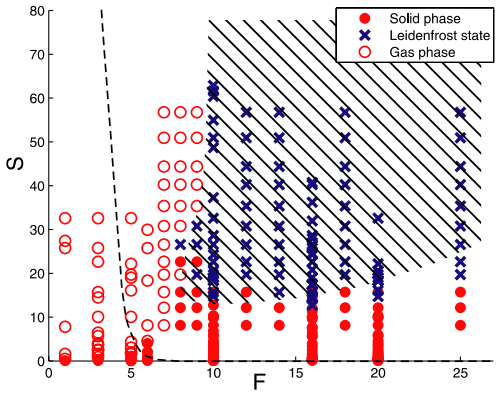

FIG. 5 (color online). Phase diagram of F (number of layers) vs S (shaking strength), with the Leidenfrost state showing up between the regions of solid and gaslike behavior. The symbols represent experimental data: solid phase (●), gas phase (○), and Leidenfrost state (×). The shaded area is the Leidenfrost regime according to our hydrodynamic theory. The dashed curve marks the onset of density inversion at the bottom, $dn/dy|_{y=0} = 0$, an essential precursor to the Leidenfrost effect.

The second equation is the energy balance between the heat flux through the vibrated bed and the dissipation due to the inelastic particle collisions:

$$\frac{d}{dy}\left\{\kappa \frac{dT}{dy} + C_1 \varepsilon l T^{3/2} \frac{dn}{dy}\right\} = \frac{\mu}{\gamma l} \varepsilon n T^{3/2}. \tag{5}$$

On the left-hand side, the thermal conductivity κ is proportional to the product of the density n, average particle velocity ($\propto T^{1/2}$, with T the granular temperature), and the mean free path l: $\kappa \propto n T^{1/2} l$ [13]. The second term on the left-hand side only becomes important when the density gradient dn/dy is large [14]. The term on the right is equal to the energy loss in one collision ($\propto \varepsilon T$) multiplied by the total number of collisions ($\propto n T^{1/2}$) [6]. The coefficients C_1, μ, and γ are constants.

Third, we have the equation of state [13,20]:

$$p = nT \frac{n_c + n}{n_c - n}, \tag{6}$$

which is the ideal-gas law ($p = nT$) corrected for excluded volume effects, with $n_c = 2/\sqrt{3}d^2$ being the number density of the close-packed hexagonal crystal. Equation (6) is an interpolation between the well-established equations of state in the low and high density limit [13,20,21].

The three equations (4)–(6) are supplemented by three boundary conditions. The first one states that the granular temperature at the bottom of the container is constant: $T_0 = \text{const}$. The second condition is that the energy flux must be zero at the top of the system: $\lim_{y\to\infty}[\kappa(y) dT/dy] = 0$, and the third condition is the conservation of particles: $\int_0^\infty n(y) dy = F n_c d$.

The above set of equations plus boundary conditions can be solved numerically, using a shooting method for the vanishing heat flux at infinity (second condition). Two typical examples of the resulting density profiles are shown in Figs. 2(c) and 2(f). They agree qualitatively with the experimental profiles, apart from the oscillations associated with the particle packing, which of course do not show up in the continuum approach. In all cases we encountered, the term proportional to dn/dy in the energy balance (5) proved to be negligible compared to the dT/dy term. So for our system Eq. (5) simplifies to:

$$\frac{d}{dy}\left\{\kappa \frac{dT}{dy}\right\} = \frac{\mu}{\gamma l} \varepsilon n T^{3/2}. \tag{7}$$

The equations (4), (6), and (7), plus boundary conditions, can be nondimensionalized by introducing the variables $\tilde{y} = y/d$, $\tilde{n} = n/n_c$, and $\tilde{T} = T/T_0$. Then the following dimensionless control parameters show up in the new set of equations and conditions: the number of layers F, the inelasticity $\varepsilon = (1 - e^2)$, and the energy input S defined in Eq. (3), just as in the experiment. Thus, from the four dimensionless control parameters identified originally [see Eq. (1) and below], only *three* remain because Γ and the dimensionless shaking amplitude A do not appear individually in the model, but only combined as $S \equiv \Gamma A$ [22].

The parameter S is proportional to the typical kinetic energy of the particles at the bottom [$\propto \frac{1}{2}m(af)^2$] divided by the potential energy needed for a particle to overcome its own diameter [mgd].

To extract the theoretical Leidenfrost regime in the (S, F) plane (the shaded area in Fig. 5) we proceed as follows from the calculated density profiles [23]: the *onset* of the Leidenfrost effect from the solid state is taken to occur when a layer of at least 2 particle diameters near the bottom drops below the density threshold $n_1 = 0.97n_c$. When S is increased beyond its critical value $S_{\text{onset}} \approx 16$, the crystalline as well as the gaslike phase gradually become more dilute. The *breakdown* of the Leidenfrost state, where it gives way to a pure gas, is taken to occur when the part of the profile exceeding $n_2 = 0.85n_c$ becomes less than 6 particle diameters thick. This upper boundary rises very steeply, e.g., at $F = 16$ layers $S_{\text{breakdown}} \approx 2000$ [24].

The required minimum thicknesses of the gaseous and the crystalline phase (2 respective 6 particle diameters) agree with the experimentally observed thicknesses in our snap shots. The parameters n_1 and n_2 also reasonably agree with the snap shots; their precise values are chosen such as to yield an optimal overlap between the experimental and theoretical Leidenfrost regimes in Fig. 5.

In Fig. 5 we have also included the dashed curve where $dn/dy|_{y=0}$ goes through zero, i.e., the onset of the density inversion studied earlier by Meerson *et al.* [6]. Above this curve $dn/dy|_{y=0}$ is positive, which paves the way for the Leidenfrost effect, but in itself does not mark a phase transition yet. Physically, the granular material is still fully either a gas or a solid at the dashed line [as can be verified via the order parameter $O(y)$].

In conclusion, the granular Leidenfrost effect has been demonstrated experimentally for the first time, in a 2D setup: when the shaking strength S exceeds a critical value, and for sufficiently many particle layers, a dense cluster with a hexagonal packing floats on top of a gaseous region. The two coexisting phases in this hybrid state (solid and gas) can be distinguished from each other by their difference in crystalline order, through the order parameter $O(y)$. The experimental observations are quantitatively explained by our hydrodynamic model, as shown by the density profiles in Fig. 2 and the (S, F) diagram of Fig. 5.

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