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# The granular Leidenfrost effect

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ABSTRACT: The granular Leidenfrost effect is experimentally observed in a vertically vibrated 2D container filled with glass beads: Above a critical shaking strength, and for a sufficient number of particles, a *cluster* of beads (with a crystalline packing) is elevated and supported by a few fast gas-like particles. This Leidenfrost state is reached through a second order phase transition. The experimental observations are confirmed by a theoretical model.

## 1 INTRODUCTION

The original "Leidenfrost effect" of a water droplet hovering over a hot plate was first reported by Hermann Boerhaave in 1732, and investigated more thoroughly by Johann Gottlob Leidenfrost, who in 1756 published "A Tract About Some Qualities of Common Water" (Leidenfrost 1756). The effect can be observed when a drop of water is put on a hot surface, above the so-called Leidenfrost temperature $T_L \approx 220°C$ (see Figure 1): The bottom layer of the drop vaporizes instantly and prevents direct heat transfer from the plate to the drop, causing the droplet to hover and survive for a long time (Walker 2001).

Here we report the first experimental observation of the *granular* Leidenfrost effect, in the quasi-2D container ($10 \times 0.45 \times 14$ cm) shown in Figure 2. The container is filled with glass beads of diameter $d = 4.0$ mm, density $\rho = 2.5$ g/cm$^3$, and coefficient of normal restitution $e \approx 0.95$. When the setup is vertically vibrated above a critical shaking strength (equivalent to heating the plate above $T_L$ in the original Leidenfrost experiment), and for a sufficient number of particles, a dense cluster of particles is elevated and supported by a dilute layer of fast particles underneath. The cluster shows an almost perfect hexagonal close packing. This clustered state was predicted theoretically by Meerson et al. (2003), and is an exceptionally strong form of the density inversion found in most granular systems driven by a vertically vibrating bottom (Lan & Rosasto 1995).

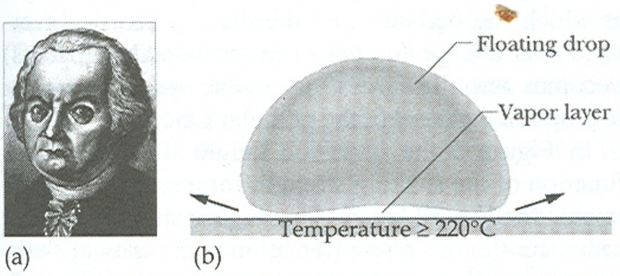


Figure 1. (a) Johann Gottlob Leidenfrost. (b) The Leidenfrost effect: A drop of water above a hot plate floating on its own vapor layer.

This paper focuses upon the transition between the solid state (for weak shaking) and the Leidenfrost state (for strong shaking), which is found to be a second-order phase transition. One of the main results is the identification of the three dimensionless control parameters that govern this transition.

## 2 SECOND ORDER PHASE TRANSITION

When one analyzes the experiments, the parameter space to be examined is considerable: Shaking amplitude $a$ and frequency $f$, diameter $d$ and normal restitution coefficient $e$ of the beads, the number of bead

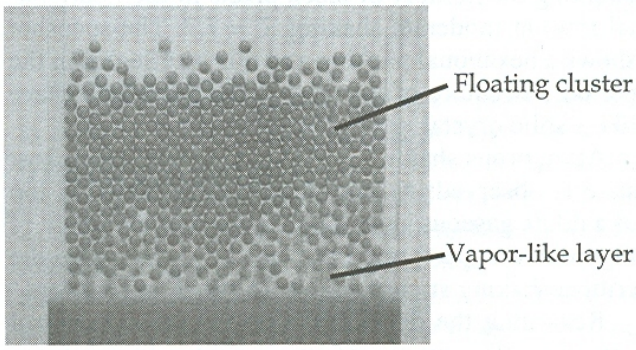


Figure 2. The granular Leidenfrost effect: Glass beads, vertically vibrated above a critical shaking strength, form a crystalline cluster that is elevated and supported by a vapor-like layer of fast particles underneath.

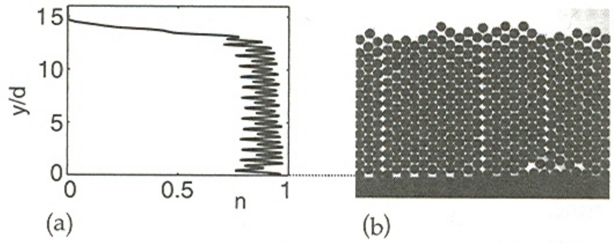


Figure 3. (a) The density profile as a function of height for $F = 16$ layers at shaking strength $\Gamma = 7.7$ ($a = 0.3$ mm, $f = 80$ Hz): The periodicity of the profile corresponds to the hexagonal packing of the particles. The origin $y/d = 0$ is set at the maximal positive displacement of the vibrating bottom. (b) A typical snapshot of this experiment recorded by a high-speed camera (1000 fps).

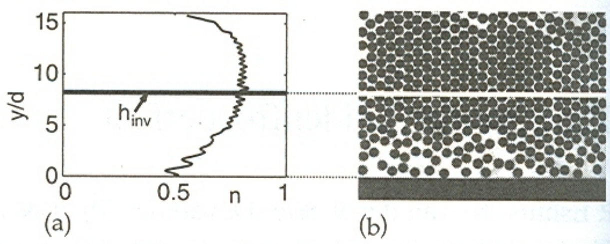


Figure 4. (a) Density profile for $F = 16$ layers at $\Gamma = 51.5$, showing the Leidenfrost state, with a crystalline phase floating on top of a gaseous layer. The inversion height $h_{inv}$ is the border between the gaseous and the solid phase, being determined as the height where the derivative of the averaged density profile (over 300 consecutive snapshots) becomes zero. (b) The corresponding experimental snapshot.

layers $F$, and others. The four natural and easy-to-vary dimensionless control parameters are the shaking strength (with $g$ the gravitational acceleration):

$$\Gamma = \frac{a(2\pi f)^2}{g}, \tag{1}$$

the number of bead layers:

$$F, \tag{2}$$

the dimensionless shaking amplitude:

$$A = \frac{a}{d}, \tag{3}$$

and finally, the inelasticity parameter:

$$\varepsilon = (1 - e^2). \tag{4}$$

First the dependence on the shaking strength $\Gamma$ is investigated for a fixed number of layers $F = 16$. Figure 3 shows an experimental snapshot and the corresponding density profile $n(y/d)$ (determined by counting the number of black pixels in each horizontal row) at moderate shaking, $\Gamma = 7.7$. The snapshot shows a hexagonal packing and this is reflected in the periodic structure of $n(y/d)$, i.e. the particles behave like a solid crystal.

At vigorous shaking, see Figure 4, the Leidenfrost state is observed: A crystalline cluster floats on top of a dilute gaseous layer. The particular experiment of Figure 4 was performed at $\Gamma = 51.5$, well above the critical shaking strength ($\Gamma_c \approx 25$) for $F = 16$ layers.

Regarding the dependence on the second control parameter (the number of layers $F$) it is found in our experiments that a Leidenfrost state only occurs for $F \geq 10$. For smaller numbers of layers one immediately goes from a solid to a granular gas, without a crystalline cluster floating on top. By adding particles to the system the granular Leidenfrost effect can be triggered.

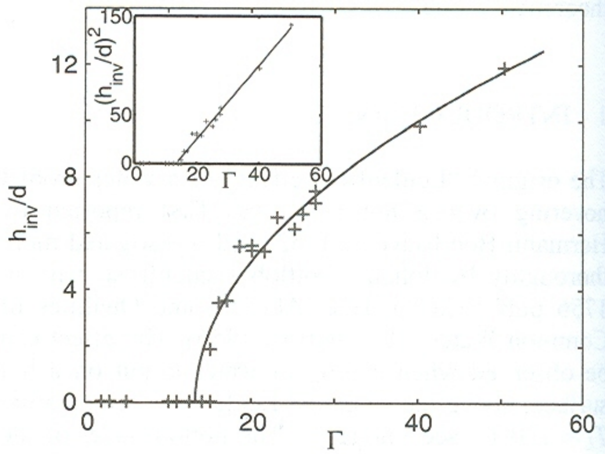


Figure 5. The dimensionless inversion height $h_{inv}/d$ as a function of the shaking strength $\Gamma$ (at a fixed frequency $f = 50$ Hz) for $F = 10$ layers, showing a second-order phase transition at $\Gamma_c = 15$. The inset demonstrates that $h_{inv}^2$ grows linearly with $(\Gamma - \Gamma_c)$, indicating that the critical exponent $\beta$ has the mean field value 1/2.

So, the Leidenfrost state is observed above a critical shaking strength $\Gamma$ and for a sufficient number of layers $F$. To distinguish between the gaseous and the crystalline phase in this state we introduce the inversion height $h_{inv}$. This quantity is defined as the height at which the derivative of the density profile (averaged over a large number of experimental snapshots) becomes zero. It serves as a suitable *order parameter* to pinpoint the onset of the granular Leidenfrost effect.

In Figure 5 the inversion height is plotted as a function of the shaking strength for a series of experiments for $F = 10$ layers. This Figure shows a second order, continuous phase transition at the critical shaking strength $\Gamma_c = 15$. As shown by the inset of Figure 5, a linear relation is found for $h_{inv}^2$ versus $\Gamma$, i.e. the inversion height grows as $h_{inv} \propto (\Gamma - \Gamma_c)^{\beta}$ with the mean field exponent $\beta = 1/2$.

## 3 HYDRODYNAMIC MODEL

In order to explain the Leidenfrost state and the associated phase transition, we use a continuum description of the granular material in the setup. Furthermore, we disregard the effect of the side walls, which makes our theory essentially one-dimensional. The model is based on three hydrodynamic equations that have been derived within the context of the kinetic theory of granular gases (Haff 1983; Grossman et al. 1997; Eggers 1999; Meerson et al. 2003).

The first one is the standard force balance:

$$\frac{dp}{dy} = -mgn, \tag{5}$$

with $p(y)$ the pressure, $m$ the mass of a single particle, and $n(y)$ the number density.

The second equation is the energy balance between the heat flux through the vibrated bed and the dissipation due to inelastic collisions:

$$\frac{d}{dy}\{\kappa\frac{dT}{dy} + C_1 \varepsilon l T^{3/2}\frac{dn}{dy}\} = \frac{\mu}{\gamma l}\varepsilon n T^{3/2}. \tag{6}$$

On the left hand side, the thermal conductivity $\kappa$ is proportional to the product of the density $n$, the average particle velocity ($\propto T^{1/2}$, with $T$ the granular temperature), and the mean free path $l$: $\kappa \propto nT^{1/2}l$ (Grossman et al. 1997). The second term on the left hand side only becomes important when the density gradient $dn/dy$ is large (Sela & Goldhirsch 1998). The term on the right hand side is equal to the energy loss in one collision ($\propto \varepsilon T$) times the total number of collisions ($\propto nT^{1/2}$) (Meerson et al. 2003). The coefficients $C_1$, $\mu$, and $\gamma$ are constants.

The third equation of the model is the equation of state (Grossman et al. 1997; Herbst et al. 2004):

$$p = nT\frac{n_c + n}{n_c - n}, \tag{7}$$

which is the ideal-gas law ($p = nT$) corrected for excluded volume effects, with $n_c = 2/\sqrt{3}d^2$ being the number density of the close-packed hexagonal crystal. Equation (7) is an interpolation between the well-established equations of state in the low and high density limit. Also other possible interpolation formulae have been put forward in recent years, see e.g. Luding (2001).

The three equations (5)–(7) are supplemented by the following three boundary conditions. The first one states that the granular temperature at the bottom of the container is constant:

$$T_0 \propto (af)^2 = \text{constant}. \tag{8}$$

The second condition is that the energy flux is zero at the top of the system (implying a constant granular temperature at $y \to \infty$):

$$\left.\frac{dT}{dy}\right|_{y\to\infty} = 0. \tag{9}$$

Conservation of the total number of particles gives us the third and last boundary condition:

$$\int_0^\infty n(y)\,dy = Fn_c d, \tag{10}$$

with $F$ the number of layers.

The above set of equations plus boundary conditions can be solved numerically, using a shooting method for the vanishing heat flux at infinity, Eq. (9). The results will be presented elsewhere (Eshuis et al. 2005) and yield density profiles similar to those obtained by Meerson et al. (2003); they show that the term proportional to $dn/dy$ in the energy balance Eq. (6) is negligible in our case. So this equation simplifies to:

$$\frac{d}{dy}\{\kappa\frac{dT}{dy}\} = \frac{\mu}{\gamma l}\varepsilon n T^{3/2}. \tag{11}$$

## 4 DIMENSIONLESS CONTROL PARAMETERS

The equations (5, 7, 11) and boundary conditions (8, 9, 10) of the hydrodynamic model can be nondimensionalized by introducing the new variables $\tilde{y} = y/d$, $\tilde{n} = n/n_c$, and $\tilde{T} = T/T_0$. We then find that the following (dimensionless) parameter combinations show up in the new set of equations and conditions: The number of layers $F$ (Eq. 2), the inelasticity $\varepsilon = (1 - e^2)$ (Eq. 4), and the energy input

$$S = \frac{(a2\pi f)^2}{gd}. \tag{12}$$

The first two of these ($F$ and $\varepsilon$) are just as defined in Section 2, but $S$ is new. It is proportional to the typical kinetic energy of the particles at the bottom [$\propto \frac{1}{2}m(af)^2$] divided by the potential energy needed for a particle to overcome its own diameter [$mgd$]. We note that $S \equiv \Gamma A$, i.e. the product of the shaking strength $\Gamma$ and the dimensionless shaking amplitude $A$. These two numbers do not appear individually in the model, but only as a combination.

To check whether $S$ also plays such a central role in experiment, we determined the critical shaking amplitude $a$ and frequency $f$ (at the transition to the Leidenfrost state) for a number of experimental runs

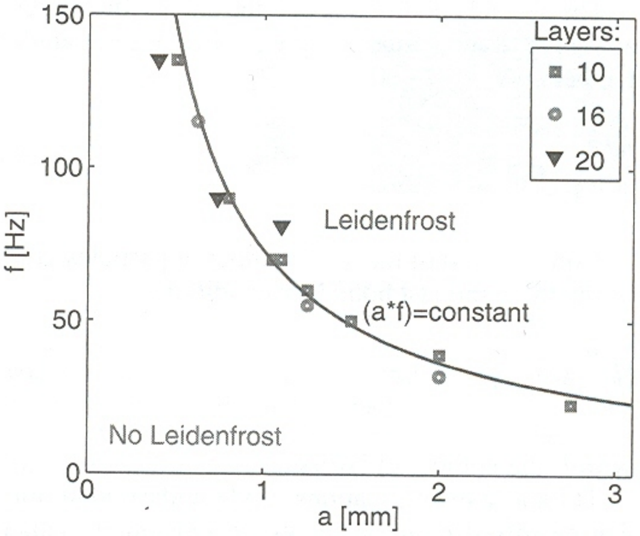


Figure 6. The critical values of the shaking amplitude $a$ and frequency $f$ at which the transition towards the Leidenfrost state occurs, for a number of experiments with $F \geq 10$ layers. The product $af$ is constant along the transition curve, or equivalently, $S \equiv \Gamma A \propto (af)^2$ is constant.

with $F \geq 10$ layers. These critical points are plotted in the $(a,f)$-plane of Figure 6.

We observe that along the curve that marks the transition towards the Leidenfrost state, the product $(af)$ is constant. This means that $S \equiv \Gamma A \propto (af)^2$ is constant at the transition. So indeed, $S$ (and not $\Gamma$) is the fundamental shaking parameter, just as in the hydrodynamic model.

The fact that $(af)^2$ is the crucial shaking parameter in vibro-fluidized granular systems was already proposed on theoretical grounds by Warr et al. (1994). Experimental and numerical work during the last decade was less conclusive, yielding $(af)^\alpha$ with the exponent $\alpha$ taking on values between 1 and 2 depending on the system under consideration (Luding et al. 1994; Warr et al. 1995; Huntley 1998; Kumaran 1998; Wildman & Huntley 2003). In this context, it is noteworthy that in the granular Leidenfrost system both theory and experiment agree on the value $\alpha = 2$.

## 5 CONCLUDING REMARKS

In conclusion, the granular Leidenfrost effect has been observed in a two-dimensional experiment for the first time: If the shaking strength exceeds a critical value, for a sufficient number of particle layers, a dense cluster with an almost perfect hexagonal packing floats on top of a gaseous region. This Leidenfrost state is reached directly from the solid state through a second order phase transition.

We find, both in experiment and theory, that the transition is governed by three dimensionless control parameters: The energy input $S$, the number of layers $F$, and the inelasticity of the particle collisions $\varepsilon$.


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